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## Liquid Pipeline Hydraulics

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## Introduction to Liquid Pipeline Hydraulics

This online course on liquid pipeline hydraulics covers the steady state transportation of liquids in pipelines. These include water lines, refined petroleum products and crude oil pipelines. This course will prove to be a refresher in fluid mechanics as it is applied to real world pipeline design. Although many formulas and equations are introduced, we will concentrate on how these are applied to the solution of actual pipeline transportation problems.

First, the liquid properties are discussed, and how they vary with temperature and pressure are analyzed. The pressures in a liquid and liquid head are explained next. Then the classical method of determining pressure drop due to friction in liquid flow is introduced and a modified more practical versions are explained. Common forms of equations relating to flow versus pressure drop due to friction are introduced, and applications are illustrated by example problems. In a long distance pipeline, the need for multiple pump stations and hydraulic pressure gradient is discussed.

Next the pumping horsepower required to transport a liquid through a pipeline is calculated. The internal design pressure in a pipeline and the hydrostatic test pressure for safe operation are explained with illustrative examples. The use of drag reduction as a means to improving pipeline throughput is explored. Centrifugal and positive displacement pumps are discussed along with an analysis of the pump performance curves. The impact of liquid specific gravity and viscosity on pump performance is explained with reference to the Hydraulic Institute charts.

## 1. Properties of Liquids

Mass is the amount of matter contained in a substance. It is sometimes interchangeably used in place of weight. Weight however, is a vector quantity and depends upon the acceleration due to gravity at the specific location. In the English or US Customary System of units (USCS). Mass and weight are generally referred to in pounds (Ib). Strictly speaking, we must refer to mass as pound-mass (lbm) and weight as pound force (lbf). Mass is independent of temperature and pressure. In SI or Metric units, mass is measured in kilograms (kg). As an example, a 55 gal drum of crude oil may weigh 412 lb or has a mass of 412 lb .

Volume is the space occupied by a particular mass. It depends upon temperature and pressure. For liquids, pressure has very little effect on volumes compared to gases. Volume of liquid increases slightly with increase in temperature. In USCS units volume may be expressed in gallons (gal) or cubic feet $\left(\mathrm{ft}^{3}\right)$. One $\mathrm{ft}^{3}$ is equal to 7.481 US gal. In the Oil and Gas industry, volume of petroleum products is measured in barrels (bbl). One bbl is equal to 42 US gal. In SI or Metric units, volume is expressed in liters ( L ) or cubic meters $\left(\mathrm{m}^{3}\right)$. One $\mathrm{m}^{3}$ is equal to 1000 L . A useful conversion is 3.785 L to a gal of liquid.

Density of a liquid is defined as mass per unit volume. Therefore, like volume, density also depends upon temperature and pressure. Density decreases with increase in liquid temperature and vice versa. In USCS units density is stated in $\mathrm{lb} / \mathrm{gal}^{\mathrm{l}} \mathrm{or} \mathrm{lb} / \mathrm{ft}^{3}$. In the petroleum industry, sometimes density is expressed in lb/bbl. In SI or Metric units, density is expressed in $\mathrm{kg} / \mathrm{L}$ or $\mathrm{kg} / \mathrm{m}^{3}$. If the mass of a 55 gal drum of crude oil is 412 lb , the density of the crude oil is $412 / 55=7.49 \mathrm{lb} / \mathrm{gal}$. In contrast water has a density of 8.33 $\mathrm{lb} / \mathrm{gal}$ or $62.4 \mathrm{lb} / \mathrm{ft}^{3}$. In SI units, the density of water is approximately $1000 \mathrm{~kg} / \mathrm{m}^{3}$ or 1
metric tonne/m ${ }^{3}$. The term specific weight is also sometimes used with liquids. It is calculated by dividing the weight by the volume.

Specific Gravity is defined as the ratio of the density of a liquid to that of water at the same temperature. It is therefore a measure of how heavy the liquid is compared to water. Being a ratio, specific gravity is dimensionless. Considering the density of water as 8.33 $\mathrm{lb} / \mathrm{gal}$ and a sample of crude oil with a density of $7.49 \mathrm{lb} / \mathrm{gal}$, we calculate the specific gravity of the crude oil as $7.49 / 8.33=0.899$. Sometimes, specific gravity is abbreviated to gravity. Since the density of a liquid changes with temperature, the specific gravity also depends on temperature. Since density decreases with temperature rise, the specific gravity also decreases with increase in temperature. For example, if the specific gravity of a petroleum product is 0.895 at $60^{\circ} \mathrm{F}$, its specific gravity at $100^{\circ} \mathrm{F}$ may be 0.815 . The variation of specific gravity with temperature is approximately linear as shown in the equation below.

$$
\begin{equation*}
S_{T}=S_{60}-a(T-60) \tag{1.1}
\end{equation*}
$$

where, $S_{T}$ is the specific gravity at temperature $T, S_{60}$ is the specific gravity at $60^{\circ} \mathrm{F}$ and a is a constant that depends on the liquid. Charts are available that show the specific gravity versus temperature variation for various liquids. See Crane Handbook (References).

In the petroleum industry, the term API Gravity is often used to describe the gravity of crude oils and refined petroleum products. It is based upon a standard of $60^{\circ} \mathrm{F}$ and API gravity of 10.0 for water. For lighter liquids such as gasoline and crude oil, the API gravity is a number higher than 10.0. Therefore, the heavier the liquid is compared to water, the lower is the API value. A typical crude oil is said to have a gravity of 27 deg API. Consider for example, gasoline with a specific gravity of 0.74 (compared to water $=1.00$ ). The corresponding API gravity of gasoline is 59.72 deg API. Similarly diesel with a specific
gravity of 0.85 , has an API gravity of 34.97. Conversion between specific gravity and API gravity can be done using the following equations:

Specific gravity $\mathrm{Sg}=141.5 /(131.5+$ API $)$
or $\quad \mathrm{API}=141.5 / \mathrm{Sg}-131.5$

Substituting API of 10.0 for water results in a specific gravity of 1.0 for water. It must be noted that API gravity is always defined at $60^{\circ} \mathrm{F}$. Therefore, the specific gravity used in the above equations must also be at $60^{\circ} \mathrm{F}$.

## Example 1

(a) Calculate the specific gravity of a crude oil that has an API gravity of 29.0.
(b) Convert a specific gravity of 0.82 to API gravity.

## Solution

(a) From equation 1.2 we get:

$$
\mathrm{Sg}=141.5 /(131.5+29.0)=0.8816
$$

(b) From equation 1.3 we get:

$$
\mathrm{API}=141.5 / 0.82-131.5=41.06
$$

Viscosity of a liquid represents the resistance to flow and is defined by the classical Newton's equation that relates the shear stress in the liquid to the velocity gradient of flow. When liquid flows through a pipeline the velocity of liquid particles at any cross-section varies in some fashion depending upon the type of flow (laminar or turbulent). Generally the particles close to the pipe wall will be at rest (zero velocity) and as we move towards the center of the pipe the velocity increases. The velocity variation may be considered to be approximately trapezoidal for turbulent flow or close to a parabola for laminar flow. Considering one half cross section of the pipe, the liquid velocity varies from zero to a
maximum of $\underline{u}_{\text {max }}$. If the distance measured from the pipe wall to the center of pipe cross section is y , the velocity gradient is $\mathrm{du} / \mathrm{dy}$. This is depicted in Figure 1.1.


Figure 1.1 Velocity Gradient

Newton's Law states that the shear stress $\tau$ between successive layers of liquid is proportional to the velocity gradient du/dy. The constant of proportionality is known as the dynamic or absolute viscosity of liquid $\mu$.

$$
\begin{equation*}
\tau=\mu \mathrm{du} / \mathrm{dy} \tag{1.4}
\end{equation*}
$$

The absolute viscosity is measured in $\mathrm{Ib} / \mathrm{ft}-\mathrm{s}$ in USCS units and in Poise ( P ) or centipoise (cP) in SI units. A related term known as the kinematic viscosity, denoted by $v$ is defined as the ratio of the absolute viscosity $\mu$ to the liquid density $\rho$ at the same temperature.

$$
\begin{equation*}
\nu=\mu / \rho \tag{1.5}
\end{equation*}
$$

In USCS units $v$ is stated in $\mathrm{ft}^{2} / \mathrm{s}$ and in SI units, it is expressed in $\mathrm{m}^{2} / \mathrm{s}$, Stokes (S) or centistokes (cSt). In dealing with petroleum products, kinematic viscosity in cSt is used in both USCS units and SI units. However, sometimes, in testing petroleum products in the laboratory kinematic viscosity is stated in units of SSU and SSF. SSU stands for Saybolt

Universal Seconds and SSF is for Saybolt Furol Seconds. SSU is used for heavy crude oils and SSF for heavy fuel oils. For example, the viscosity of Alaskan North Slope Crude (ANS) is 200 SSU at $60^{\circ} \mathrm{F}$. Viscosity conversion from SSU and SSF to cSt and vice versa can be done using the following formulas:

$$
\begin{align*}
\text { cSt }= & 0.226(\mathrm{SSU})-195 /(\mathrm{SSU})  \tag{1.6}\\
& \text { for } 32 \leq \mathrm{SSU} \leq 100 \\
\mathrm{cSt}= & 0.220(\mathrm{SSU})-135 /(\mathrm{SSU})  \tag{1.7}\\
& \text { for } \mathrm{SSU}>100 \\
\mathrm{cSt}= & 2.24(\mathrm{SSF})-184 /(\mathrm{SSF})  \tag{1.8}\\
& \text { for } 25<\mathrm{SSF} \leq 40 \\
\mathrm{cSt}= & 2.16(\mathrm{SSF})-60 /(\mathrm{SSF})  \tag{1.9}\\
& \text { for } \mathrm{SSU}>40
\end{align*}
$$

For example, a viscosity of 200 SSU can be converted to cSt as follows:

$$
\text { cSt }=0.220 \times 200-135 / 200=43.33
$$

Similarly, viscosity of 200 SSF can be converted to cSt as follows:

$$
\text { cSt }=2.16 \times 200-60 / 200=431.7
$$

It can be observed from the equations above that converting kinematic viscosity from SSU and SSF into cSt is fairly easy. However, to convert from cSt to SSU or SSF is not straight forward. You will have to solve a quadratic equation. This will be illustrated in the example below. A rule of thumb is that the SSU value is approximately 5 times the cSt value.

## Example 2

Convert a viscosity of 150 cSt to SSU.

Assuming the viscosity in SSU will be approximately $5 \times 150=750$, we can then use equation 1.7 as follows:

$$
150=0.220 \times \text { SSU }-135 / \text { SSU }
$$

Transposing we get a quadratic equation in SSU:
$0.22 x^{2}-150 x-135=0$
where x is viscosity in SSU
Solving for x , we get:
$x=682.72 \mathrm{SSU}$

Similar to density and specific gravity, the viscosity of a liquid decreases with increase in temperature. However, the variation is not linear. For example, the viscosity of a crude oil at $60^{\circ} \mathrm{F}$ is 40 cSt and at $100^{\circ} \mathrm{F}$, the viscosity decreases to 20 cSt . The viscosity temperature variation is approximately as follows:

$$
\begin{equation*}
\log _{e}(v)=A-B(T) \tag{1.10}
\end{equation*}
$$

where $v$ is the viscosity in cSt at an absolute temperature $T$. The absolute temperature is measured in degrees Rankin (deg R) in USCS units or Kelvin (K) in SI units. A and B are constants. The absolute temperature scale is defined as follows:

In USCS units, $\operatorname{deg} R=\operatorname{deg} F+460$.
In SI units, $K=\operatorname{deg} C+273$.

It can be seen from Equation 1.10 that a plot of $\log$ (viscosity) versus absolute temperature $T$ is a straight line. Generally, from laboratory data, the viscosity of a crude oil or petroleum product is given at two different temperatures. From this data a plot of viscosity versus temperature can be made on a special graph paper known as ASTM D341. Once we plot the two sets of points, the viscosity at any intermediate temperature can be determined by interpolation.

Vapor Pressure of a liquid is defined as that pressure at a certain temperature when the liquid and its vapor are in equilibrium. Consequently, the boiling point of a liquid is the temperature at which its vapor pressure equals the atmospheric pressure. In a laboratory, the liquid vapor pressure is always measured at $100^{\circ} \mathrm{F}$ and referred to as the Reid Vapor Pressure (RVP). The vapor pressure of a liquid increases with increase in temperature. Therefore if the Reid Vapor pressure of a liquid 10.0 psig, the vapor pressure at $70^{\circ} \mathrm{F}$ will be a lower number such as 8.0 psig. The actual vapor pressure of a liquid at any temperature may be obtained from charts, knowing its Reid vapor pressure. The vapor pressure of a liquid is important in determining the minimum suction pressure available for a centrifugal pump. This is discussed in further detail under centrifugal pumps.

Bulk Modulus of a liquid is a measure of the compressibility of the liquid. It is defined as the pressure necessary to cause a unit change in volume. Generally, liquids are considered practically incompressible, compared to gases:

$$
\begin{equation*}
\text { Bulk modulus }=\frac{\Delta P}{\Delta V / V} \tag{1.11}
\end{equation*}
$$

In differential form,

$$
\begin{equation*}
\mathrm{K}=\quad V(d P / d V) \tag{1.12}
\end{equation*}
$$

where $\Delta \mathrm{V}$ is the volume change for a pressure change of $\Delta \mathrm{P}$.

For water, K is approximately 300,000 psig whereas for gasoline it is 150,000 psig. It can be seen from the values of $K$ that fairly large pressures are required to cause small volume changes in liquids. In USCS units bulk modulus K is stated in psig. Two types of K values are used: isothermal and adiabatic. The following formulas are used to calculate the isothermal and adiabatic bulk modulus.

The adiabatic bulk modulus of a liquid of given API gravity and at pressure $P$ psig and temperature $\mathrm{T} \operatorname{deg} \mathrm{R}$ is:

$$
\begin{equation*}
\mathrm{Ka}=\mathrm{A}+\mathrm{B}(\mathrm{P})-\mathrm{C}(\mathrm{~T})^{1 / 2}-\mathrm{D}(\mathrm{API})-\mathrm{E}(\mathrm{API})^{2}+\mathrm{F}(\mathrm{~T})(\mathrm{API}) \tag{1.13}
\end{equation*}
$$

where constants A through F are defined as follows:
$A=1.286 \times 10^{6}$
$B=13.55$
$C=4.122 \times 10^{4}$
$D=4.53 \times 10^{3}$
$E=10.59$
$F=3.228$

The isothermal bulk modulus of a liquid of given API gravity and at pressure P psig and temperature $T \operatorname{deg} R$ is:

$$
\begin{equation*}
\mathrm{Ki}=\mathrm{A}+\mathrm{B}(\mathrm{P})-\mathrm{C}(\mathrm{~T})^{1 / 2}+\mathrm{D}(\mathrm{~T})^{3 / 2}-\mathrm{E}(\mathrm{API})^{3 / 2} \tag{1.14}
\end{equation*}
$$

where constants A through E are defined as follows:

$$
\begin{array}{lll}
A=2.619 \times 10^{6} & B=9.203 & \\
C=1.417 \times 10^{5} & D=73.05 & E=341.0
\end{array}
$$

## Example 3

Calculate the two values of bulk modulus of a liquid with an API gravity of 35 , at 1,000 psig pressure and $80^{\circ} \mathrm{F}$.

## Solution

The adiabatic bulk modulus is:
$K a=1.286 \times 10^{6}+13.55(1000)-4.122 \times 10^{4}(80+460)^{1 / 2}-4.53 \times 10^{3}(35)-10.59(35)^{2}+$ $3.228(80+460)(35)$

Solving for the adiabatic bulk modulus, we get:

$$
\mathrm{Ka}=231,170 \mathrm{psig}
$$

Similarly, The isothermal bulk modulus is:

$$
\mathrm{Ki}=2.619 \times 10^{6}+9.203 \times 1000-1.417 \times 10^{5}(80+460 \mathrm{~T})^{1 / 2}+73.05 \times(80+460 \mathrm{~T})^{3 / 2}-341.0
$$

$$
x(35)^{3 / 2}
$$

Substituting the values of A through $E$, we get:

$$
\mathrm{Ki}=181,450 \mathrm{psig}
$$

## 2. Pressure Drop Due To Friction

Pressure at any point, in a liquid is the force per unit area. Considering a body of liquid such as in a storage tank, the pressure at a depth of H feet below the liquid surface is the same at all points in the liquid in all directions. This is known as Pascal's Law. As the depth increases, the liquid pressure also increases. In USCS units, pressure is measured in $\mathrm{lb} / \mathrm{in}^{2}$ or psi. In SI units, pressure is stated in kilopascal (kPa), megapascal (MPa) or Bar.

When the atmospheric pressure at a location is included, the pressure is referred to as the absolute pressure. The pressure measured by a pressure gauge is known as the gauge pressure and does not include the atmospheric pressure. The relationship between the gauge pressure and absolute pressure is as follows:

Absolute pressure $=$ gauge pressure + atmospheric pressure

Absolute pressure is stated in psia while the gauge pressure is represented as psig. Similarly, in SI units, pressure is expressed either kPa absolute or kPa gauge.

At sea level, in USCS units the atmospheric pressure is approximately 14.7 psia. Therefore the absolute pressure in a pipeline where the pressure gauge reading is $1,000 \mathrm{psig}$ is:

$$
P_{\mathrm{abs}}=1000+14.7=1,014.7 \text { psia }
$$

In SI units, the atmospheric pressure at sea level is 101 kPa . Therefore, a gauge pressure of $5,000 \mathrm{kPa}$ is equal to an absolute pressure of:

$$
P_{\mathrm{abs}}=5000+101=5,101 \mathrm{kPa} \text { absolute } .
$$

Some conversions between USCS and SI units should be noted:
$1 \mathrm{psi}=6.895 \mathrm{kPa}$
$1 \mathrm{Bar}=100 \mathrm{kPa}=14.5 \mathrm{psi}$
$1 \mathrm{MPa}=1,000 \mathrm{kPa}=145 \mathrm{psi}$

Consider liquid contained in a tank with the liquid level at a height of H feet above the tank bottom. Due to the density of the liquid, all points within the liquid at the bottom of the tank will experience a pressure due to the column of liquid of height $H$. At the surface of the liquid the pressure is equal to the atmospheric pressure. If a pressure gauge is used to measure the liquid pressure at the tank bottom, it will register the pressure equivalent to the liquid height H . This is referred to as the liquid head when expressed in ft . The conversion between the liquid head H in ft and pressure P in psi , is related by the liquid specific gravity Sg as follows:

$$
\begin{equation*}
\text { Pressure, } \mathrm{P}=\mathrm{H} \times \mathrm{Sg} / 2.31 \tag{2.2}
\end{equation*}
$$

Thus a liquid head of 100 ft is equivalent to a pressure of:

$$
P=100 \times 1.0 / 2.31=43.3 \text { psi for water }
$$

and

$$
P=100 \times 0.74 / 2.31=32.03 \text { psi for gasoline }(\mathrm{Sg}=0.74)
$$

It is clear that the lighter the liquid, the lower the pressure in psi for the same liquid head in ft.

From a given pressure P in psi we may also calculate the equivalent liquid head H in ft , using Equation 2.2 as follows:

$$
\begin{equation*}
\mathrm{H}=2.31 \times \mathrm{P} / \mathrm{Sg} \tag{2.3}
\end{equation*}
$$

Therefore, a pressure of 500 psi may be converted to the equivalent head for each liquid as follows:

$$
\mathrm{H}=2.31 \times 500 / 1.0=1,155 \mathrm{ft} \text { for water }
$$

and $\quad H=2.31 \times 500 / 0.74=1,561 \mathrm{ft}$ for gasoline
It can be seen that for a given pressure in psi, the equivalent head in ft will increase with decrease in specific gravity. This is because the lighter liquid has to rise to a higher level to equal the given pressure in psi compared to the heavier liquid.

The concept of pressure in psi and liquid head in ft may be further illustrated by considering a pipeline which has a liquid pressure of 500 psi as shown in Figure 2.1.


Figure 2.1 Liquid Pressure in a Pipeline

It can be seen that the pressure gauge reading of 500 psi equates to a manometric head of $1,561 \mathrm{ft}$ when the liquid is gasoline. Replacing the liquid with water will create a manometric head of $1,155 \mathrm{ft}$.

In most liquid hydraulics calculations, pressure is stated in psig as measured using a pressure gauge. However, when dealing with centrifugal pumps the pressure generated by the pump is stated in feet of liquid head. Also in the basic pressure drop equation known as the Darcy equation, the term head loss is used as will be explained shortly.

Consider a pipeline of inside diameter D (inch) and length $\mathrm{L}(\mathrm{ft})$ in which a liquid of specific gravity Sg flows from point A to point B at a flow rate of Q gal/min. If the flow rate is steady at every cross-section of the pipe such as A, B or C, the same amount of liquid flows per minute. This means that the liquid molecules move at the same average velocity at $A, B$ or
C. Since the diameter is constant, this uniform velocity can be calculated from the flow rate as follows:

$$
\begin{equation*}
\text { Velocity }=\text { Flow rate/ area of flow } \tag{2.4}
\end{equation*}
$$

After some simplification, the velocity V in $\mathrm{ft} / \mathrm{s}$ is given by:

$$
\begin{equation*}
V=0.4085 \mathrm{Q} / \mathrm{D}^{2} \tag{2.5}
\end{equation*}
$$

In the above, the flow rate Q is in $\mathrm{gal} / \mathrm{min}$. When Q is in $\mathrm{bbl} / \mathrm{h}$, the velocity becomes:

$$
\begin{equation*}
V=0.2859 Q / D^{2} \tag{2.6}
\end{equation*}
$$

The corresponding velocity in SI units, is calculated as follows:

$$
\begin{equation*}
V=353.6777 Q / D^{2} \tag{2.7}
\end{equation*}
$$

where $Q$ is in $\mathrm{m}^{3} / \mathrm{h}, \mathrm{D}$ is in mm and V is in $\mathrm{m} / \mathrm{s}$

It must be remembered, however, that the liquid molecules at any cross section have velocities ranging from zero at the pipe wall to a maximum at the centerline of the pipe as discussed earlier in Figure 1.1. The velocity variation approximates a parabola at low flow rates (laminar flow) and resembles a trapezium at high flow rates (turbulent flow). The Equations 2.4 through 2.7 above give the average velocity at any cross section.

For example, consider gasoline flowing through a pipeline with an inside diameter 15.5 inch at a flow rate of 5,000 bbl/h. The average velocity can be calculated as:

$$
V=0.2859 \times 5000 /(15.5)^{2}=5.95 \mathrm{ft} / \mathrm{s}
$$

Similarly, water flowing through a 394 mm inside diameter pipe at $800 \mathrm{~m}^{3} / \mathrm{h}$ has an average velocity using Equation 2.7 as follows:

$$
V=353.6777 \times 800 /(394)^{2}=1.82 \mathrm{~m} / \mathrm{s}
$$

The Reynolds number of flow is a non-dimensional parameter that characterizes the flow as laminar or turbulent. The Reynolds number depends upon the liquid velocity, viscosity and pipe diameter. It is calculated from the following equation:

$$
\begin{equation*}
\mathrm{R}=\mathrm{VD} / \mathrm{V} \tag{2.8}
\end{equation*}
$$

where the velocity $V$ is in $f t / s$, pipe inside diameter $D$ is in $f t$, and the liquid viscosity $v$ is in $\mathrm{ft}^{2} / \mathrm{s}$. With these units R is a dimensionless parameter.

Using common pipeline units, the Reynolds number equation becomes:

$$
\begin{equation*}
\mathrm{R}=2214 \mathrm{Q} /(v \mathrm{D}) \tag{2.9}
\end{equation*}
$$

where $Q$ is the flow rate in $b b l / h, D$ is the inside diameter of pipe in inches and $v$ is the kinematic viscosity in cSt.

When Q is in $\mathrm{bbl} /$ day:

$$
\begin{equation*}
\mathrm{R}=92.24 \mathrm{Q} /(v \mathrm{D}) \tag{2.10}
\end{equation*}
$$

When Q is in gal/min, the Reynolds number equation becomes:

$$
\begin{equation*}
\mathrm{R}=3160 \mathrm{Q} /(\mathrm{vD}) \tag{2.11}
\end{equation*}
$$

In SI units, the Reynolds number is calculated from:

$$
\begin{equation*}
\mathrm{R}=353,678 \mathrm{Q} /(\mathrm{vD}) \tag{2.12}
\end{equation*}
$$

where $Q$ is in $\mathrm{m}^{3} / \mathrm{h}, \mathrm{D}$ is in mm , and $v$ is in cSt.

## Example 4

Calculate the Reynolds number of flow for a crude oil pipeline, 20 inch outside diameter and 0.500 inch wall thickness at a flow rate of $200,000 \mathrm{bbl} / \mathrm{day}$. Viscosity of crude oil is 15 cSt .

## Solution

The Reynolds number is calculated from Equation 2.10 as follows:

$$
R=92.24(200000) /(15 \times 19.0)=64,723
$$

Depending upon the value of $R$ calculated from above equations, the flow may be categorized as laminar flow, critical flow or turbulent flow as follows:

Laminar flow for $R<=2,000$
Critical flow for $R>2,000$ and $R<4,000$
Turbulent flow for $\quad R>4,000$

The upper limit for laminar flow is sometimes stated as 2,100 in some publications. Laminar flow is defined as steady state flow in which the liquid flows through the pipe smoothly in laminations. This type of flow is also known as low friction or viscous flow in which no eddies or turbulence exist. As the flow rate increases, more and more disturbance or eddies are formed due to friction between the adjacent layers of the liquid as well as friction between the pipe wall and the liquid. Due to friction, the pressure in the liquid decreases from the inlet of the pipe to the outlet.

The amount of pressure loss due to friction, also known as head loss depends upon many factors. The classical equation for head loss due to friction in pipe flow is the Darcy equation expressed as follows:

$$
\begin{equation*}
h=f(L / D) V^{2} / 2 g \tag{2.13}
\end{equation*}
$$

In the above equation the head loss $h$ is in $f t$ of liquid head. The pipe length $L$ and inside diameter D are both in ft . The velocity of flow V is in $\mathrm{ft} / \mathrm{s}$. The constant g is the acceleration due to gravity and is equal to $32.2 \mathrm{ft} / \mathrm{s}^{2}$. The parameter f is known as the Darcy friction factor that depends upon the internal roughness of the pipe, the Reynolds number R and the inside diameter of the pipe. Its value ranges from approximately 0.008 to 0.10 depending upon many factors. For laminar flow, f depends only on R. In turbulent flow, f also depends upon the internal roughness of the pipe and pipe inside diameter.

In laminar flow, the friction factor $f$ is calculated from:

$$
\begin{equation*}
f=64 / R \tag{2.14}
\end{equation*}
$$

Thus for a Reynolds number of 1,800 , the flow being laminar, the friction factor is calculated as:

$$
f=64 / 1800=0.0356
$$

If the velocity of flow in this case is $1.5 \mathrm{ft} / \mathrm{s}$ in a pipe 13.5 inch inside diameter and considering a pipe length of $1,000 \mathrm{ft}$, we can use the Darcy equation to calculate the head loss due to friction from Equation 2.13 as:

$$
\begin{aligned}
& h=f(L / D) V^{2} / 2 g \\
& h=0.0356(1000 \times 12 / 13.5)(1.5)^{2} /(2 \times 32.2)=1.106 \mathrm{ft}
\end{aligned}
$$

If the liquid specific gravity is 0.74 we can convert this head loss into psig as follows:
Pressure drop due to friction $=1.106 \times 0.74 / 2.31=0.35 \mathrm{psig}$

The calculation of the friction factor $f$ used in the Darcy equation for turbulent flow is more complex and will be discussed next. The Colebrook-White equation below, is used to calculate the friction factor in turbulent flow:

$$
\begin{equation*}
1 / \sqrt{ } \mathrm{f}=-2 \log _{10}[(\mathrm{e} / 3.7 \mathrm{D})+2.51 /(\mathrm{R} \sqrt{ } \mathrm{f})] \tag{2.15}
\end{equation*}
$$

where $f$ is the Darcy friction factor, $D$ is the pipe inside diameter and $e$ is the absolute pipe roughness. Both $D$ and $e$ are in inches. $R$ is the dimensionless Reynolds number. In SI units the same equation can be used as long as both $D$ and $e$ are in mm .

It is clear that the equation for $f$ has to be solved by trial and error since $f$ appears on both sides of the equation. An initial value of $f$ (such as 0.02 ) is assumed and substituted into the right hand side of the equation. A second approximation can then be calculated, which in turn can be used to obtain a better value of $f$ and so on. The iteration is stopped when successive values of $f$ are within a small value such as 0.001 .

## Example 5

Calculate the friction factor for a Reynolds number of 50,000 for flow in an NPS 20 pipe, 0.500 inch wall thickness and an internal roughness of 0.002 inch.

## Solution

The NPS 20 pipe has an inside diameter:

$$
\begin{aligned}
& D=20-2 \times 0.500=19.0 \text { inch } \\
& 1 / \sqrt{ } f=-2 \log _{10}[(0.002 /(3.7 \times 19.0))+2.51 /(50000 \sqrt{ } f)]
\end{aligned}
$$

Assume $\mathrm{f}=0.02$ and calculate the next approximation as:

$$
\begin{aligned}
& 1 / \sqrt{ } f=-2 \log _{10}[(0.002 /(3.7 \times 19.0))+2.51 /(50000 \sqrt{ } 0.02)] \\
& =6.8327
\end{aligned}
$$

Therefore, $\mathrm{f}=0.0214$ as the second approximation.
Using this value the next approximation for $f$ is:

$$
\begin{aligned}
1 / \sqrt{ } \mathrm{f} & =-2 \log _{10}[(0.002 /(3.7 \times 19.0))+2.51 /(50000 \sqrt{ } 0.0214)] \\
& =6.8602
\end{aligned}
$$

Therefore, $f=0.0212$ which is close enough.

In the critical zone where the Reynolds number is between 2000 and 4000, the flow is unstable and the friction factor is calculated considering turbulent flow using the ColebrookWhite equation discussed above.

Another method of calculating the friction factor in turbulent flow is using the Moody Diagram shown in Figure 2.2


Figure 2.2 Moody Diagram

The Moody Diagram is used to read off the value of friction factor, knowing the Reynolds number plotted on the horizontal axis and the relative roughness (e/D) plotted on the vertical axis to the right. It is a quick way of determining the friction factor in turbulent flow without resorting to the trial and error approach using the Colebrook-White equation.

Similar to the non-dimensional friction factor $f$, another parameter $F$ known as the transmission factor is used in pipeline hydraulics. The dimensionless term, transmission factor has an inverse relationship to the friction factor as indicated below:

$$
\begin{equation*}
F=2 / \sqrt{ } f \tag{2.16}
\end{equation*}
$$

Generally, the friction factor $f$ has a value between 0.008 and 0.10 . Therefore, using the above equation we can deduce that the transmission factor F will have values approximately in the range of 6 to 22 . It can be stated that since the pressure drop due to friction is proportional to the friction factor, it will decrease with increase in the value of the transmission factor.

The Darcy equation discussed earlier is not convenient for pipeline calculations since it does not employ common pipeline units, such as pressure in psi and flow in $\mathrm{bbl} / \mathrm{h}$. Therefore, the following equation is generally used when dealing with pipelines transporting crude oils and refined petroleum products. Pressure drop $P_{m}$, in psi per mile of pipe length, is calculated for a given flow rate, pipe inside diameter, liquid specific gravity and the friction factor from:

$$
\begin{equation*}
P_{\mathrm{m}}=0.2421(\mathrm{Q} / \mathrm{F})^{2} \mathrm{Sg} / \mathrm{D}^{5} \tag{2.17}
\end{equation*}
$$

where Q is the flow rate in $\mathrm{bbl} /$ day and F is the dimensionless transmission factor. The liquid specific gravity is Sg and the pipe inside diameter D is in inches.

In terms of the friction factor $f$, the equation for pressure drop becomes:

$$
\begin{equation*}
P_{m}=0.0605 f Q^{2}\left(S g / D^{5}\right) \tag{2.18}
\end{equation*}
$$

When the flow rate is in $\mathrm{bbl} / \mathrm{h}$ and $\mathrm{gal} / \mathrm{min}$, the equations for $\mathrm{P}_{\mathrm{m}}$ are as follows:

$$
\begin{array}{ll}
P_{\mathrm{m}}=139.45(\mathrm{Q} / \mathrm{F})^{2} \mathrm{Sg} / \mathrm{D}^{5} & \text { for } \mathrm{Q} \text { in } \mathrm{bbl} / \mathrm{h} \\
\mathrm{P}_{\mathrm{m}}=284.59(\mathrm{Q} / \mathrm{F})^{2} \mathrm{Sg} / \mathrm{D}^{5} & \text { for } \mathrm{Q} \text { in } \mathrm{gal} / \mathrm{min} \tag{2.20}
\end{array}
$$

In SI units the pressure drop due to friction $\mathrm{P}_{\mathrm{km}}$ expressed in $\mathrm{kPa} / \mathrm{km}$ is calculated from the following equations:

$$
\begin{align*}
& P_{\mathrm{km}}=6.2475 \times 10^{10} \mathrm{f} \mathrm{Q}^{2}\left(\mathrm{Sg} / \mathrm{D}^{5}\right)  \tag{2.21}\\
& \mathrm{P}_{\mathrm{k} m}=24.99 \times 10^{10}(\mathrm{Q} / \mathrm{F})^{2}\left(\mathrm{Sg} / \mathrm{D}^{5}\right) \tag{2.22}
\end{align*}
$$

where the liquid flow rate Q is in $\mathrm{m}^{3} / \mathrm{h}$ and the pipe inside diameter D is in mm .

Remember in all the equations $F$ and $f$ are related by Equation 2.16. Since $F$ and $f$ are inversely related, we can state that the higher the transmission factor, the higher will be the throughput Q. In contrast, the higher the friction factor, the lower the value of Q. This flow rate is directly proportional to f .

## Example 6

Calculate the pressure drop per mile in an NPS 20 pipeline, 0.500 inch wall thickness flowing diesel $(\mathrm{Sg}=0.85$ and viscosity $=5.0 \mathrm{cSt})$ at $8,000 \mathrm{bbl} / \mathrm{h}$. Assume pipe absolute roughness $=0.002$ inch.

## Solution

First calculate the Reynolds number:

$$
R=92.24(8000 \times 24) /(5.0 \times 19.0)=186,422
$$

Since the flow is turbulent, we can use Colebrook-White equation or the Moody diagram to determine the friction factor.

The relative roughness $=(e / D)=0.002 / 19=0.000105$. For this value of $(e / D)$ and $R=$ 186,422 we get the friction factor f from the Moody diagram as follows:

$$
f=0.0166
$$

Therefore, the transmission factor is:

$$
F=2 /(0.0166)^{1 / 2}=15.52
$$

The pressure drop per mile is:

$$
\begin{aligned}
& P_{\mathrm{m}}=139.45(8000 / 15.52)^{2} \times 0.85 /(19.0)^{5} \\
& =12.72 \mathrm{psi} / \mathrm{mi}
\end{aligned}
$$

The Hazen-Williams equation is commonly used in hydraulic analysis of water pipelines. It is used to calculate the pressure drop in a water pipeline given the pipe diameter and flow rate, taking into account a the internal condition of the pipe using the dimensionless parameter C. This parameter is called the Hazen-Williams C factor and is a function of the internal roughness of the pipe. Unlike the friction factor, the C factor increases with the smoothness of the pipe. In this regard it is more comparable to the transmission factor F discussed earlier. Values of C range from 60 to 150 or more depending upon the pipe material and roughness as indicated in Table 2.1

| Pipe Material | C-factor |
| :--- | :--- |
| Smooth Pipes (All metals) | $130-140$ |
| Smooth Wood | 120 |
| Smooth Masonry | 120 |
| Vitrified Clay | 110 |
| Cast Iron (Old) | 100 |
| Iron (worn/pitted) | $60-80$ |
| Polyvinyl Chloride (PVC) | 150 |
| Brick | 100 |

## Table 2.1 Hazen-Williams C factors

Although the Hazen-Williams equation is mostly used in water pipelines, today many companies use it to calculate the pressure drop in pipelines transporting refined petroleum products such as gasoline and diesel. For example, when used with water pipelines a $C$ value of 110 or 120 may be used and with gasoline and diesel typical values for C are 150 and 125, respectively. It must be noted that historically the value of $C$ used is based on
experience with the particular liquid and pipeline and, therefore, varies from pipeline to pipeline and with the company. However, the Colebrook-White equation is also used for most liquids and a comparison can be made with Hazen-Williams equation as illustrated in an example later. The classical form of the Hazen-Williams equation for pressure drop in water pipelines is as follows:

$$
\begin{equation*}
h=4.73 \mathrm{~L}(\mathrm{Q} / \mathrm{C})^{1.852} / \mathrm{D}^{4.87} \tag{2.23}
\end{equation*}
$$

where $h$ is the head loss due to friction in ft of water. The pipe length $L$ and diameter $D$ are both in ft and the flow rate Q is in $\mathrm{ft}^{3} / \mathrm{s}$.

A more commonly used version of the Hazen-Williams equation is as follows:

$$
\begin{equation*}
\mathrm{Q}=6.7547 \times 10^{-3}(\mathrm{C})(\mathrm{D})^{2.63}(\mathrm{~h})^{0.54} \tag{2.24}
\end{equation*}
$$

where Q is in $\mathrm{gal} / \mathrm{min}, \mathrm{D}$ is in inches, and h is the head loss due to friction in ft of liquid per 1000 ft of pipe.

Another version of the Hazen-Williams equation in common pipeline units is as follows:

$$
\begin{equation*}
Q=0.1482(C)(D)^{2.63}\left(P_{m} / S g\right)^{0.54} \tag{2.25}
\end{equation*}
$$

where Q is in $\mathrm{bbl} /$ day, D is in inches, and $\mathrm{P}_{\mathrm{m}}$ is the pressure drop due to friction in $\mathrm{psi} / \mathrm{mi}$ of pipe. The specific gravity Sg is included so that it can be used for liquids other than water. In SI Units, the Hazen-Williams formula is as follows:

$$
\begin{equation*}
\mathrm{Q}=9.0379 \times 10^{-8}(\mathrm{C})(\mathrm{D})^{2.63}\left(\mathrm{P}_{\mathrm{km}} / \mathrm{Sg}\right)^{0.54} \tag{2.26}
\end{equation*}
$$

where $Q$ is in $\mathrm{m}^{3} / \mathrm{h}, \mathrm{D}$ is in mm , and $\mathrm{P}_{\mathrm{km}}$ is the pressure drop due to friction in $\mathrm{kPa} / \mathrm{km}$.

## Example 7

Calculate the pressure drop per mile in an NPS 20 pipeline, 0.500 inch wall thickness flowing water at $7,500 \mathrm{gal} / \mathrm{min}$. Use Hazen-Williams equation and $\mathrm{C}=120$.

## Solution

Substituting the given values in Equation 2.24, the head loss $h$ can be calculated as follows:

$$
7500=6.7547 \times 10^{-3}(120)(19.0)^{2.63}(h)^{0.54}
$$

Solving for $h$, we get:

$$
\mathrm{h}=13.09 \mathrm{ft} / 1000 \mathrm{ft} \text { of pipe }
$$

Therefore, the pressure drop per mile, $\mathrm{P}_{\mathrm{m}}$ is:

$$
P_{m}=(13.09 \times 1.0 / 2.31) \times(5280 / 1000)=29.92 \mathrm{psig} / \mathrm{mi}
$$

For comparison, we will also calculate the pressure drop using the Colebrook-White equation as follows:

First calculate the Reynolds number, considering the viscosity of water as 1.0 cSt .

$$
R=3160 \times 7500 /(1.0 \times 19.0)=1,247,368
$$

Since the flow is turbulent we can use the Colebrook-White equation or the Moody diagram to determine the friction factor. We will also assume an absolute pipe roughness of 0.002 in. The relative roughness $=(e / D)=0.002 / 19=0.000105$. For this value of (e/D) and $R=$ $1,247,368$, we get the friction factor from the Moody diagram as follows:

$$
f=0.0135
$$

Therefore, the transmission factor is:

$$
F=2 /(0.0135)^{1 / 2}=17.21
$$

The pressure drop per mile is:

$$
\begin{aligned}
P_{\mathrm{m}} & =284.59(7500 / 17.21)^{2} 1.0 /(19.0)^{5} \\
& =21.83 \mathrm{psi} / \mathrm{mi}
\end{aligned}
$$

So far we have discussed the two popular pressure drop formulas used in the pipeline industry. The Colebrook-White equation is applicable for all liquids over a wide range of Reynolds numbers. The Hazen-Williams equation originally developed for water pipelines is
now used with refined petroleum products as well. Several other pressure drop equations are used in the crude oil and petroleum business and it would be useful to summarize them here.

The Miller equation is used in crude oil pipelines and does not consider the Reynolds number or pipe roughness. It requires an iterative solution to calculate the pressure drop $\mathrm{P}_{\mathrm{m}}$ from a given flow rate, liquid properties and pipe diameter. The most common form of the Miller equation is as follows:

$$
\begin{equation*}
\mathrm{Q}=4.06(\mathrm{M})\left(\mathrm{D}^{5} \mathrm{P}_{\mathrm{m}} / \mathrm{Sg}\right)^{0.5} \tag{2.27}
\end{equation*}
$$

The parameter $M$ is defined as follows:

$$
\begin{equation*}
M=\log _{10}\left(D^{3} S g P_{m} / c p^{2}\right)+4.35 \tag{2.28}
\end{equation*}
$$

where the flow rate Q is in $\mathrm{bbl} /$ day and the liquid viscosity cP is in centipoise. All other symbols have been defined before.

It can be seen that $M$ depends on $P_{m}$ and is also found in the equation connecting $Q$ and $P_{m}$. Therefore, from a given value of pressure drop $P_{m}, M$ can be calculated and $Q$ can be found using equation 2.27. However, if $P_{m}$ is unknown, to calculate $P_{m}$ from a given value of flow rate, pipe diameter and liquid properties, we must use a trial and error approach for solving for $P_{m}$ using the intermediate parameter $M$. The method is to assume a value of $P_{m}$ and calculate the corresponding value of $M$. Substituting this value of $M$ in Equation 2.28 we can solve for a new value of $\mathrm{P}_{\mathrm{m}}$. This forms the second approximation which can then be used to calculate a new value of $M$ and then a better value of $P_{m}$. The process is repeated until successive values of $P_{m}$ are within 0.001 .

In SI Units, the Miller Equation is expressed as follows:

$$
\begin{equation*}
\mathrm{Q}=3.996 \times 10^{-6}(\mathrm{M})\left(\mathrm{D}^{5} \mathrm{P}_{\mathrm{m}} / \mathrm{Sg}\right)^{0.5} \tag{2.29}
\end{equation*}
$$

And the parameter $M$ is defined as follows:

$$
\begin{equation*}
M=\log _{10}\left(D^{3} S g P_{m} / c p^{2}\right)-0.4965 \tag{2.30}
\end{equation*}
$$

where all items have been defined previously.

The MIT equation developed jointly by Shell and MIT, sometimes known as Shell-MIT equation is used in heavy crude oil pipelines that are heated to reduce the viscosity and enhance pipe flow. It considers pipe roughness and uses a modified Reynolds number. The Reynolds number is first calculated and then a modified Reynolds number obtained by dividing it by 7742 as follows:

$$
\begin{align*}
& \mathrm{R}=92.24(\mathrm{Q}) /(\mathrm{D} v)  \tag{2.31}\\
& \mathrm{Rm}=\mathrm{R} /(7742) \tag{2.32}
\end{align*}
$$

where the flow rate Q is in $\mathrm{bbl} /$ day and Viscosity $v$ is in cSt .

Depending upon the type of flow (laminar or turbulent) a friction factor is calculated using the following equations:

$$
\begin{align*}
& f=0.00207 / R m \text { (for laminar flow) }  \tag{2.33}\\
& f=0.0018+0.00662(1 / R m)^{0.355} \text { (for turbulent flow) } \tag{2.34}
\end{align*}
$$

The friction factor f calculated above is not the same as the Darcy friction factor f calculated using the Colebrook-White equation.

The pressure drop due to friction, $\mathrm{P}_{\mathrm{m}}$ is then calculated from:

$$
\begin{equation*}
P_{m}=0.241\left(f \mathrm{SgQ}^{2}\right) / \mathrm{D}^{5} \tag{2.35}
\end{equation*}
$$

All symbols have been defined previously.
In SI Units, the MIT Equation is stated as follows:

$$
\begin{equation*}
P_{m}=6.2191 \times 10^{10}\left(f \mathrm{SgQ}^{2}\right) / D^{5} \tag{2.36}
\end{equation*}
$$

where all symbols have been defined previously.

## Example 8

Calculate the pressure drop per mile in an NPS 16 pipeline, 0.250 inch wall thickness flowing crude oil ( $\mathrm{Sg}=0.895$ and viscosity $=15.0 \mathrm{cSt}$ ) at $100,000 \mathrm{bbl} /$ day. Assume pipe absolute roughness $=0.002$ inch. Compare the results using the Colebrook-White, Miller and MIT equations.

## Solution

Pipe inside diameter $D=16.0-2 \times 0.250=15.50 \mathrm{in}$.
Next, calculate the Reynolds number:

$$
R=92.24 \times 100000 /(15.0 \times 15.50)=39,673
$$

Since the flow is turbulent we use the Colebrook-White equation in the first case to determine the friction factor.

The relative roughness $=(e / D)=0.002 / 15.5=0.000129$

$$
1 / \sqrt{ } f=-2 \log _{10}[(0.000129 / 3.7)+2.51 /(39673 \sqrt{ } f)]
$$

Solving for $f$ by trial and error we get:

$$
f=0.0224
$$

The pressure drop due to friction using the Colebrook-White equation is:

$$
\begin{aligned}
& P_{\mathrm{m}}=0.0605 \times 0.0224 \times(100000)^{2}\left(0.895 /(15.5)^{5}\right) \\
& =13.56 \mathrm{psi} / \mathrm{mi}
\end{aligned}
$$

For the Miller equation we assume $\mathrm{P}_{\mathrm{m}}=14 \mathrm{psi} / \mathrm{mi}$ as the first approximation. Then calculating the parameter M as follows:

$$
M=\log _{10}\left((15.5)^{3} 0.895 \times 14 /(15 \times 0.895)^{2}\right)+4.35
$$

where viscosity in $c P=15 \times 0.895=13.425$
Therefore, $\mathrm{M}=6.7631$
Next calculate the value of $\mathrm{P}_{\mathrm{m}}$ from Equation 2.27:

$$
100000=4.06 \times 6.7631 \times\left[(15.5)^{5} \mathrm{P}_{\mathrm{m}} / 0.895\right]^{0.5}
$$

Solving for $\mathrm{P}_{\mathrm{m}}$, we get:

$$
P_{m}=13.27 \mathrm{psi} / \mathrm{mi}, \text { compared to the assumed value of } 14.0
$$

Recalculating $M$ from this latest value of $P_{m}$, we get:

$$
M=\log _{10}\left((15.5)^{3} 0.895 \times 13.27 /(15 \times 0.895)^{2}\right)+4.35
$$

Therefore, $M=6.74$
Next calculate the value of $P_{m}$ from Equation 2.27:

$$
100,000=4.06 \times 6.74 \times\left[(15.5)^{5} \mathrm{P}_{\mathrm{m}} / 0.895\right]^{0.5}
$$

Solving for $P_{m}$, we get:

$$
P_{\mathrm{m}}=13.36 \mathrm{psi} / \mathrm{mi}
$$

Repeating the iteration once more, we get the final value of the pressure drop with the Miller Equation as:

$$
\mathrm{P}_{\mathrm{m}}=13.35 \mathrm{psi} / \mathrm{mi}
$$

Next we calculate the pressure drop using MIT equation.
The modified Reynolds number is:

$$
R m=R / 7742=39673 / 7742=5.1244
$$

Since $\mathrm{R}>4,000$, the flow is turbulent and we calculate the MIT friction factor from Equation
2.34 as:

$$
f=0.0018+0.00662(1 / 5.1244)^{0.355}=0.0055
$$

The pressure drop due to friction is calculated from Equation 2.35:

$$
\begin{aligned}
& P_{\mathrm{m}}=0.241 \times 0.0055 \times 0.895 \times(100000)^{2} /(15.5)^{5} \\
& =13.32 \mathrm{psi} / \mathrm{mi}
\end{aligned}
$$

Therefore, in summary the pressure drop per mile using the three equations are as follows:

$$
\begin{array}{ll}
\mathrm{P}_{\mathrm{m}}=13.56 \mathrm{psi} / \mathrm{mi} & \text { using Colebrook-White equation } \\
\mathrm{P}_{\mathrm{m}}=13.35 \mathrm{psi} / \mathrm{mi} & \text { using Miller equation } \\
\mathrm{P}_{\mathrm{m}}=13.32 \mathrm{psi} / \mathrm{mi} & \text { using MIT equation }
\end{array}
$$

Finally it is seen that in this case all three pressure drop equations yield approximately the same value for $\mathrm{psi} / \mathrm{mi}$ with the Colebrook-White equation being the most conservative (highest pressure drop for given flow rate).

Using the previously discussed equations such as Colebrook-White, Hazen-Williams, etc. we can easily calculate the frictional pressure drop in a straight piece of pipe. Many appurtenances such as valves and fittings installed in pipelines also contribute to pressure loss. Compared to several thousand feet (or miles) of pipe, the Pressure Losses through fittings and valves are small. Therefore, such pressure drops through valves, fitting and other appurtenances are referred to as minor losses. Such losses may be calculated in a couple of different ways.

Using the equivalent length concept, the valve or fitting is said to have the same frictional pressure drop as that of a certain length of straight pipe. Once the equivalent length of the device is known, the pressure drop in that straight length of pipe can be calculated. For example, a gate valve is said to have an equivalent length to the diameter ratio of 8 . This means that a 16 inch gate valve has the same amount of pressure drop as a straight piece of a 16 inch pipe with a length of $8 \times 16$ or 128 inches. Therefore, to calculate the pressure drop in a 16 inch gate valve at $5,000 \mathrm{bbl} / \mathrm{h}$ flow rate we would simply calculate the $\mathrm{psi} / \mathrm{mi}$ value in a 16 inch pipe and use proportion as follows:

Let pressure drop in the 16 inch pipe $=12.5 \mathrm{psi} / \mathrm{mi}$
Pressure drop in the 16 inch gate valve $=12.5 \times 128 /(5280 \times 12)=0.0253 \mathrm{psi} / \mathrm{mi}$
It can be seen that the minor loss through a gate valve is indeed small in comparison with, say, 1000 ft of 16 inch pipe ( $12.5 \times 1000 / 5280=2.37 \mathrm{psi})$.

Table 2.2 shows the equivalent length to pipe diameter ratio L/D for various valves and fittings.

| Description | L/ D |
| :--- | :--- |
| Gate Valve | 8 |
| Globe valve | 340 |
| Ball valve | 3 |
| Swing check valve | 50 |
| Standard Elbow - $90^{\circ}$ | 30 |
| Standard Elbow - 45 |  |
| Long Radius Elbow $-90^{\circ}$ | 16 |

Table 2.2 Equivalent length to pipe diameter ratio

The second approach to calculating the minor losses is using the K factor method. According to the Darcy equation the head loss due to friction is proportional to the velocity head $\mathrm{V}^{2} / 2 \mathrm{~g}$. Therefore, the minor loss through valves and fittings may be represented by $\mathrm{K}\left(\mathrm{V}^{2} / 2 \mathrm{~g}\right)$. The coefficient K depends upon the particular device such as valves, fitting etc. Typical K values are listed in Table 2.3

| Description | Nominal Pipe Size - inches |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1/2 | 3/4 | 1.0 | $\begin{aligned} & 1- \\ & 1 / 4 \end{aligned}$ | $\begin{gathered} 1- \\ 1 / 2 \end{gathered}$ | 2 | $\begin{gathered} 21 / 2 \text { to } \\ 3 \end{gathered}$ | 4 | 6 | $\begin{gathered} 8 \text { to } \\ 10 \end{gathered}$ | $\begin{gathered} 12 \text { to } \\ 16 \end{gathered}$ | $\begin{gathered} 18 \text { to } \\ 24 \end{gathered}$ |
| Gate Valve | 0.22 | 0.20 | 0.18 | 0.18 | 0.15 | 0.15 | 0.14 | 0.14 | 0.12 | 0.11 | 0.10 | 0.10 |
| Globe Valve | 9.2 | 8.5 | 7.8 | 7.5 | 7.1 | 6.5 | 6.1 | 5.8 | 5.1 | 4.8 | 4.4 | 4.1 |
| Ball Valve | 0.08 | 0.08 | 0.07 | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 |
| Butterfly Valve |  |  |  |  |  | 0.86 | 0.81 | 0.77 | 0.68 | 0.63 | 0.35 | 0.30 |
| Plug Valve Straightway | 0.49 | 0.45 | 0.41 | 0.40 | 0.38 | 0.34 | 0.32 | 0.31 | 0.27 | 0.25 | 0.23 | 0.22 |
| Plug Valve 3 way thruflo | 0.81 | 0.75 | 0.69 | 0.66 | 0.63 | 0.57 | 0.54 | 0.51 | 0.45 | 0.42 | 0.39 | 0.36 |
| Plug Valve branch - flo | 2.43 | 2.25 | 2.07 | 1.98 | 1.89 | 1.71 | 1.62 | 1.53 | 1.35 | 1.26 | 1.17 | 1.08 |

Table 2.3 Typical K values
It must be noted that the K factor method is applicable only for turbulent flow in valves and fittings. This is because all experimental data on minor losses using the K factor is available only for turbulent flow.

## 3. Pressure Required to Transport

In the previous section we discussed various equations to calculate the pressure drop due to friction in a pipeline transporting a liquid. For example in Example 6, we calculated that in an NPS 20 pipeline transporting diesel at a flow rate of $8,000 \mathrm{bbl} / \mathrm{h}$ the pressure drop due to friction was $12.72 \mathrm{psi} / \mathrm{mi}$. If the pipeline was 50 miles long, the total pressure drop due to friction will be $12.72 \times 50=636 \mathrm{psi}$.


Figure 3.1 Total pressure required to pump liquid

Consider now that the pipeline originates at Point $A$ and terminates at Point $B, 50$ miles away. Suppose the delivered product at Point B is required to be at a minimum pressure of 50 psi to account for pressure drop in the delivery tank farm and the tank head. If the ground elevation is essentially flat, the total pressure required at the origin of the pipeline, A is $636+50=686$ psi. The pressure of 686 psi at $A$ will drop to 50 psi at $B$ due to the friction in the 50 mile length of pipe as shown in Figure 3.1. If the ground profile was not flat, and the elevation of $A$ is 100 ft and that at $B$ is 500 ft , additional pressure is needed at A to overcome the elevation difference of (500-100) ft. Using the head to pressure
conversion equation, the 400 ft elevation difference translates to $400 \times 0.85 / 2.31$ or 147.2 psi, considering the specific gravity of diesel as 0.85 . This elevation component amounting of 147.2 psi must then be added to the 686 psi resulting in a total pressure of 833.2 psi at $A$ in order to deliver the diesel at the terminus B at 50 psi pressure. This is illustrated in Figure 3.2 .


Figure 3.2 Components of total pressure

Thus we conclude that the total pressure required to transport a liquid from Point A to Point B consists of three different components:

1. Friction Head
2. Elevation Head
3. Minimum Delivery Pressure

A graphical representation of the pressure variation along the pipeline from Point $A$ to Point $B$ is depicted in Figure 3.3 and is known as the Hydraulic pressure gradient.


Figure 3.3 Hydraulic Pressure Gradient with peak

Since the liquid pressure in the pipeline is shown along with the pipe elevation profile, it is customary to plot the pressures in ft of liquid head instead of pressure in psi. At any point along the pipeline, the liquid pressure is represented by the vertical intercept between the hydraulic gradient and the pipeline elevation at that point. This is shown at ED in Figure 3.3. Of course, the pressure ED is in ft of liquid head and can be converted to psi, using the specific gravity of the liquid.

In addition to the elevation difference between the origin $A$ and the terminus $B$, there may be drastic elevation changes along the pipeline, with peaks and valleys. In this case, we must also ensure that the liquid pressure in the pipeline at any location does not fall below
zero (or some minimum value) at the highest elevation points. This is illustrated in Figure 3.3 where the peak in pipeline elevation at $C$ shows the minimum pressure $P_{\min }$ to be maintained.

The minimum pressure depends upon the vapor pressure of the liquid at the flowing temperature. For water, crude oils and refined petroleum products, since vapor pressures are low and we are dealing with gauge pressures, a zero gauge pressure (14.7 psia) at the high points can be tolerated. However, most companies prefer some non-zero gauge pressure at the high points such as 10 to 20 psig. For highly volatile liquids with high vapor pressures such as LPG, the minimum pressure along the pipeline must be maintained at some number such as 200 to 250 psig to prevent vaporization and consequent two phase flow. As the liquid flows through the pipeline, its pressure decreases due to friction and increases or decreases depending upon the elevation change along the various points in the pipeline profile. At some point such as $C$ in Figure 3.3, the elevation is quite high and therefore the pressure in the pipeline has dropped to a small value ( $\mathrm{P}_{\mathrm{min}}$ ) indicated by the vertical intercept between the hydraulic gradient and the pipeline elevation at point $C$. If the pressure at C drops below zero psig, vaporization of the liquid occurs and results in an undesirable situation in liquid flow.

Due to the complexity of thermal hydraulics, we will restrict our analysis to steady state isothermal flow only. This means that a constant flowing temperature of liquid is assumed throughout the pipeline. Thermal flow occurs when the liquid temperature in the pipeline varies from inlet to outlet. An example of thermal flow is the transportation of a heated heavy crude oil. Due to high viscosity, the crude oil is heated before pumping to reduce its viscosity and improve pipeline throughput. Therefore, at the beginning of the pipeline, the liquid may be heated to $140^{\circ} \mathrm{F}$ and pumped through the pipeline. As the heated liquid flows through the pipeline, heat is transferred from the liquid to the surrounding soil (buried
pipeline) or ambient air (above ground pipeline) since the surrounding soil (or ambient) is at a lower temperature than the liquid in the pipeline. This heat transfer results in the liquid temperature dropping from the inlet temp of $140^{\circ} \mathrm{F}$ as it flows the length of the pipeline, closely approaching the ambient soil temperature as shown in Figure 3.4.


Figure 3.4 Temperature variation in thermal flow

It can be seen that due to the temperature change along the pipeline, the liquid properties such as specific gravity and viscosity also vary resulting in change in the Reynolds number and pressure drop, even with the flow remaining constant. Thus the pressure drop ( $\mathrm{P}_{\mathrm{m}}$ ) in a thermal flow situation varies along the pipeline compared to the constant pressure drop in isothermal flow. A typical hydraulic gradient for thermal flow is shown in Figure 3.5 for comparison.


Figure 3.5 Hydraulic pressure gradient - isothermal and thermal flow

In the preceding discussions, we considered constant flow throughout the pipeline. Whatever volume of liquid entered the pipeline, the same amount exited the pipeline at the terminus. In the real world of pipeline transportation, it is common to find volumes of liquid entering the pipeline (injection) and exiting the pipeline (deliveries) at various points along the pipeline as shown in Figure 3.6


Figure 3.6 Injection \& Deliveries

Liquid enters the pipeline at Point $A$ at a flow rate $Q_{1}$ and at some intermediate Point $C$ a certain volume $\mathrm{Q}_{2}$ is delivered out of the pipeline. The remaining volume $\left(\mathrm{Q}_{1}-\mathrm{Q}_{2}\right)$ continues on until at Point $D$ a new volume of $Q_{3}$ enters the pipeline. The resultant volume $\left(Q_{1}-Q_{2}+Q_{3}\right)$ then continues to the terminus $B$ where it is delivered out of the pipeline. The pressure drops in each section of the pipeline such as $A C, C D$ and $D B$ must be calculated by considering the individual flow rates and liquid properties in each section. Since flow $Q_{2}$ is delivered at C, the pipe sections AC and CD have the same liquid properties but different flow rates for pressure drop calculations. The last section DB will have a different flow rate and different liquid properties due to the combination of two different streams of liquid at Point D. The total pressure required at A will be calculated by adding the pressure drop due to friction for pipe sections $A C, C D$ and $D B$ and also adding the pressure required for pipe elevation changes and including the minimum delivery pressure at the pipeline terminus.

## Series and Parallel Pipes

Pipes are said to be in series when pipe of different diameters and lengths are connected end to end and the liquid flows from the inlet of the first pipe to the outlet of the last pipe. If there are no intermediate injections or deliveries, pipes in series will have flow as a common parameter in all pipes. The total pressure required to pump a certain volume of liquid through series pipes can be calculated by simply adding the pressure drop due to friction in all pipe sections and including the elevation component and delivery pressure component as before. This is illustrated in Figure 3.7


Figure 3.7 Pipes in series

Pipes are said to be in Parallel when the flow in the first section of a pipeline splits into two or more streams flowing through different pipe sections and later rejoining into a single pipe as shown in Figure 3.8. This is also referred to as looping a pipeline.


Figure 3.8 Pipes in parallel

Liquid enters the pipeline at Point $A$ at flow rate $Q$ and at some intermediate Point $C$ the flow divides into two streams $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ flowing in two different pipes CED and CFD as shown in the Figure 3.8

The two streams later recombine at Point $D$ and the combined flow $Q=Q_{1}+Q_{2}$, then continues through the pipe $D B$ to be delivered out of the pipeline at the terminus $B$. The pressure drop in sections AC and DB can be calculated easily considering the flow rate of Q and pipe diameters and liquid properties. $C$ and $D$ are the common junction points of pipe segment CED and CFD and cause a common pressure drop for each of the two pipe segments. Knowing the value of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$, we can calculate the common pressure drop between $C$ and $D$. For this we need two equations to calculate $Q_{1}$ and $Q_{2}$. The first equation is known as the conservation of flow at the junction C or D written as follows:

$$
\begin{equation*}
\mathrm{Q}=\mathrm{Q}_{1}+\mathrm{Q}_{2} \tag{3.1}
\end{equation*}
$$

The other equation connecting $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ comes from the common pressure drop between C and $D$ applied to each of the two pipes CED and CFD. Using the Darcy equation in the modified form as indicated in Equation 2.17 we can write the following relationship for the two pressure drops:

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{CED}}=\left(\mathrm{Q}_{1} / \mathrm{F}_{1}\right)^{2} \mathrm{SgL}_{1} /\left(\mathrm{D}_{1}\right)^{5} \tag{3.2}
\end{equation*}
$$

where subscript 1 applies to the pipe section CED of length $L_{1}$.
Similarly, for pipe section CFD of length $L_{2}$ the pressure drop is:

$$
\begin{equation*}
\Delta \mathrm{P}_{\mathrm{CFD}}=\left(\mathrm{Q}_{2} / \mathrm{F}_{2}\right)^{2} \mathrm{SgL}_{2} /\left(\mathrm{D}_{2}\right)^{5} \tag{3.3}
\end{equation*}
$$

Since both pressure drops are equal in parallel flow, and assuming the transmission factors $F_{1}$ and $F_{2}$ are approximately equal, we can create the second equation between $Q_{1}$ and $Q_{2}$ as follows:

$$
\begin{equation*}
\left(Q_{1}\right)^{2} L_{1} /\left(D_{1}\right)^{5}=\left(Q_{2}\right)^{2} L_{2} /\left(D_{2}\right)^{5} \tag{3.4}
\end{equation*}
$$

Using equations 3.1 and 3.4 we can solve for $Q_{1}$ and $Q_{2}$. The common pressure drop between $C$ and $D$ can then be calculated using the equation for $\Delta P_{C E D}$. We can then calculate the total pressure drop by adding the individual pressure drops for pipe segment $A C, D B$ and $\Delta \mathrm{P}_{\text {Ced. }}$. Of course, the elevation component must also be taken into account along with the delivery pressure at $B$.

## 4. Pump Stations and Horsepower Required

In the previous sections we calculated the total pressure required to transport a certain volume of liquid through a pipeline from point $A$ to point $B$. If the pressure required is $P$ at a flow rate of Q , a pump will be needed at the origin $A$ to provide this pressure. This pump will be driven by an electric motor or engine that will provide the necessary horsepower (HP). The pump HP required can be calculated from the pressure $P$ and flow rate $Q$ as follows:

$$
\begin{equation*}
\mathrm{HP}=\mathrm{Q} \times \mathrm{P} / \text { Constant } \tag{4.1}
\end{equation*}
$$

The constant will depend upon the units employed. In addition, since the pump is not $100 \%$ efficient, we will need to take the efficiency into account to calculate the HP. Using common pipeline units, if the pressure $P$ is in psi and flow rate $Q$ in $b b l / d a y$, the pump $H P$ required is given by:

$$
\begin{equation*}
H P=Q \times P /(58776 \times \text { Effy }) \tag{4.2}
\end{equation*}
$$

where the pump efficiency Effy is a decimal value less than 1.0. This HP is also called the pump brake horsepower (BHP).

Strictly speaking, the pressure generated by the pump will be the difference between liquid pressure on the suction side of the pump (Psuct) and the pump discharge pressure (Pdisch). Therefore $P$ in the HP equation must be replaced with (Pdisch - Psuct). This is called the differential pressure generated by the pump.

## Example 9

The suction and discharge pressures at a pump station are 50 psi and 875 psi , respectively, when the liquid flow rate is $4,200 \mathrm{bbl} / \mathrm{h}$. If the pump efficiency is $85 \%$, calculate the pumping HP required.

## Solution

## From Equation 4.2

$$
B H P=(4200 \times 24) \times(875-50) /(58776 \times 0.85)
$$

When dealing with centrifugal pumps, the pressure developed by the pump is referred to as pump head and expressed in ft of liquid head. The flow rate through the pump, also known as pump capacity, is stated in gal/min. Most pump companies use water as the liquid to test the pump, and the pump performance curves are therefore plotted in terms of water. The HP equation discussed earlier can be modified in terms of pump head H in ft and flow rate Q in $\mathrm{gal} / \mathrm{min}$ for an efficiency of E (a decimal value) as follows:

$$
\begin{equation*}
\text { Pump HP }=\mathrm{Q} \times \mathrm{H} \times \mathrm{Sg} /(3960 \times \mathrm{E}) \tag{4.3}
\end{equation*}
$$

where the liquid specific gravity is Sg .

Suppose the pump develops a head of $2,500 \mathrm{ft}$ at a flow rate of $1,800 \mathrm{gal} / \mathrm{min}$ with an efficiency of $82 \%$, the pump HP required with water is calculated as:

$$
H P=1800 \times 2500 \times 1.0 /(3960 \times 0.82)=1,386
$$

If the pump is driven by an electric motor, the closest driver HP required in this case is 1,500 HP. It must be noted, that the motor efficiency would determine the actual electrical power in Kilowatts (kW) required to run the $1,500 \mathrm{HP}$ motor that is used to drive the pump that requires $1,386 \mathrm{HP}$.

In a short pipeline such as the one discussed in an earlier example, the 50 mile pipeline transporting diesel at $8,000 \mathrm{bbl} / \mathrm{h}$ required an originating pressure of 686 psi at A. Suppose the pipeline was 120 miles long from Ashton to Beaumont and calculations show that at $8,000 \mathrm{bbl} / \mathrm{h}$ flow rate, the originating pressure at Ashton is $1,576 \mathrm{psi}$. If the pipe material was strong enough to withstand the discharge pressure of 1,576 psi at Ashton, only one
pump station at Ashton will be needed to transport diesel at this flow rate from Ashton to Beaumont. However, if the pipe material was such that the maximum allowable operating pressure (MAOP) is limited to 1,440 psi, we would not be able to operate at 1,576 psi discharge at Ashton. Therefore, by limiting the discharge pressure to the MAOP value, the diesel throughput will fall below the $8,000 \mathrm{bbl} / \mathrm{h}$ value. If the throughput must be maintained at $8,000 \mathrm{bbl} / \mathrm{h}$ while limiting pipeline pressures to MAOP, we can accomplish this by installing a second pump station at some point between Ashton and Beaumont as shown in Figure 4.1


Figure 4.1 Multiple pump stations

The intermediate pump station at Hampton is called the booster pump station and will be located for hydraulic balance such that the same amount of energy is input into the liquid at each pump station. In other words, the total HP required will be equally distributed between the Ashton pump station and Hampton pump station. Since the flow rate is constant throughout the pipeline (no intermediate injections or deliveries) hydraulic balance would
imply that each pump station discharges at approximately the same pressure as shown in Figure 4.1.

Neglecting for the moment elevation difference along the pipeline and assuming the minimum suction pressure at each pump station and the delivery pressure at Beaumont to be equal to 50 psi , we can calculate the discharge pressure at each of the pump stations as follows:

$$
\text { Pdisch }+(\text { Pdisch }-50)=1576
$$

or, Pdisch $=813$ psi.

Therefore, each pump station will be operating at a discharge pressure of 813 psi. Comparing this with the pipeline MAOP of 1,440 psi, we see that there is considerable room to increase the discharge pressure and hence the flow rate. The maximum pipeline flow rate will then occur when each pump station discharges at a pressure of $1,440 \mathrm{psi}$. It can be seen that by increasing the discharge pressure to 1,440 psi causes the hydraulic gradient to be steeper than that at 813 psi which corresponds to $8,000 \mathrm{bbl} / \mathrm{h}$. If Hampton pump station is located at a distance of 60 mi from Ashton, we can calculate the pressure drop due to friction corresponding to the steeper gradient when discharging at $1,440 \mathrm{psi}$.

Since the hydraulic gradient is plotted in ft of head, the slope of the hydraulic gradient represents the frictional head loss in ft/mi, which can also be expressed in psi/mi. From this $\mathrm{psi} / \mathrm{mi}$, we can calculate the flow rate using Colebrook-White Equation as follows:

$$
P_{m}=(1440-50) / 60=23.17 \mathrm{psi} / \mathrm{mi}
$$

This is the maximum slope of the hydraulic gradient when each pump station operates at MAOP of 1,440 psig and elevation profile is neglected.

To calculate the flow rate from a given pressure drop we have to use a trial and error approach. First calculate an approximate flow rate using the fact that $\mathrm{P}_{\mathrm{m}}$ is proportional to square of the flow rate Q from Equation 2.17 . At $8,000 \mathrm{bbl} / \mathrm{h}$ the pressure drop is 12.72 $\mathrm{psi} / \mathrm{mi}$ and at the unknown flow rate $\mathrm{Q}, \mathrm{P}_{\mathrm{m}}=23.17$ :
$23.17 / 12.72=(Q / 8000)^{2}$
Solving for Q we get:

$$
\mathrm{Q}=10,797 \mathrm{bbl} / \mathrm{h}
$$

Next using this flow rate we calculate the Reynolds number and friction factor using the Colebrook-White Equation:

$$
R=92.24(10797 \times 24) /(5.0 \times 19.0)=251,600
$$

Since the flow is turbulent we can use the Colebrook-White Equation or the Moody diagram to determine the friction factor.

The relative roughness $=(e / D)=0.002 / 19=0.000105$
For this value of (e/D) and $R=251,600$ we get the friction factor $f$ from the Moody diagram as follows:

$$
f=0.0159
$$

Therefore, the transmission factor is:

$$
F=2 /(0.0159)^{1 / 2}=15.86
$$

The pressure drop per mile is therefore calculated from Equation 2.9:

$$
\begin{aligned}
& P_{\mathrm{m}}=139.45(10797 / 15.86)^{2} \times 0.85 /(19.0)^{5} \\
& =22.19 \mathrm{psig} / \mathrm{mi}
\end{aligned}
$$

Compare this with $23.17 \mathrm{psi} / \mathrm{mi}$ calculated from the pressure gradient. Therefore, the flow rate has to be increased slightly higher to produce the required pressure drop.

Next approximate the flow rate, using proportions, since flow rate is proportional to the square root of the pressure drop.

$$
\mathrm{Q}=10797(23.17 / 22.19)^{1 / 2}=11,033 \mathrm{bbl} / \mathrm{h}
$$

Recalculating the Reynolds number and friction factor we finally get the required flow rate as $11,063 \mathrm{bbl} / \mathrm{h}$. Thus, in the two pump station configuration the maximum flow rate possible without exceeding MAOP is $11,063 \mathrm{bbl} / \mathrm{h}$. The pump HP required depends upon the liquid properties such as gravity and viscosity. It can be seen from Equation 4.3 that the pump HP is directly proportional to liquid specific gravity. The effect of viscosity can be explained by realizing that the pressure drop with a high viscosity liquid is generally higher than when pumping a low viscosity liquid such as water. Therefore the pressure required to pump a high viscosity liquid is greater than that required with a low viscosity liquid. It follows then that the HP required to pump a heavy product will also be higher. When we discuss centrifugal pump performance curves in the next section, the effect of viscosity will be explained in more detail.

The hydraulic horsepower represents the pumping HP when the efficiency is considered to be $100 \%$. The Brake horsepower (BHP), on the other hand is the actual HP demanded by the pump taking into account the pump efficiency.

## Pipeline System Head Curve

In the previous sections we calculated the total pressure required at the beginning of a pipeline to transport a liquid at a certain flow rate to the pipeline terminus at a specified delivery pressure. This pressure at the pipeline origin will increase as the flow rate is increased, and hence we can develop a curve showing the pipeline pressure versus flow rate for a particular pipeline and liquid pumped. This curve is referred to as the pipeline System Head Curve or simply the System Curve for this specific product. Generally the pressure is plotted in psi or ft of liquid head. The latter units for pressure is used when dealing with centrifugal pumps. Figure 4.2 shows typical System Head Curves (in psi) for a pipeline considering two products, gasoline and diesel. It can be seen that at any flow rate the pressure required for diesel is higher than that for gasoline.


Flow Rate

Figure 4.2 Pipeline System Head Curves

## Example 10

Develop a system head curve for the 50 mi pipeline described in Example 6 for diesel flow rates ranging from $2,000 \mathrm{bbl} / \mathrm{h}$ to $10,000 \mathrm{bbl} / \mathrm{h}$.

## Solution

At each flow rate from $\mathrm{Q}=2,000 \mathrm{bbl} / \mathrm{h}$ to $\mathrm{Q}=10,000 \mathrm{bbl} / \mathrm{h}$, in increments of $2,000 \mathrm{bbl} / \mathrm{h}$, we can calculate the Reynolds number $\mathrm{P}_{\mathrm{m}}$ and then calculate the originating pressure as we did at $\mathrm{Q}=8,000 \mathrm{bbl} / \mathrm{h}$. Elevation profile of the pipeline has been neglected. The results are as follows:

| Q | 2,000 | 4,000 | 6,000 | 8,000 | 10,000 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}_{\mathrm{t}}$ | 102 | 230 | 426 | 686 | 1011 |

The system head curve is plotted below:


## 5. Pump Analysis

Pumps are used to produce the pressure required to transport a liquid through a pipeline. Both centrifugal pumps and reciprocating pumps are used in pipeline applications. Reciprocating pumps, also known as positive displacement or PD pumps produce high pressures at a fixed flow rate depending upon the geometry of the pump. Centrifugal pumps on the other hand are more flexible and provide a wider range of flow rates and pressures. Centrifugal pumps are more commonly used in pipeline applications and will be discussed in more detail. A typical centrifugal pump performance curve is shown in Figure 5.1.


Flow rate - gal/min

Figure 5.1 Centrifugal pump performance curve

There are four curves that comprise the performance of a particular centrifugal pump. These are as follows:

1. Head versus Flow Rate
2. Efficiency versus Flow Rate
3. BHP versus Flow Rate
4. NPSH versus Flow Rate

Pump vendors use the term capacity when referring to the flow rate. It can be seen that in a typical centrifugal pump, the head (or differential pressure) generated by the pump decreases as the flow rate (also known as pump capacity) increases. This is known as a drooping head versus flow characteristic. The efficiency, on the other hand, increases as the flow rate increases and reaches a peak value (known as the best efficiency point or BEP) and falls off rapidly with further increase in flow rate. The BHP also increases with increase in flow rate. As mentioned before, centrifugal performance curves are generally plotted considering water as the liquid pumped. Therefore, the BHP is calculated considering specific gravity $=1.0$.

The NPSH curve also increases with increase in flow rate. The term NPSH refers to Net Positive Suction Head and is a measure of the minimum suction head required at the suction of the pump impeller at any flow rate. NPSH is a very important parameter when dealing with centrifugal pumps particularly when pumping volatile liquids. It will be discussed in more detail later.

When pumping highly viscous liquids the centrifugal pump performance curves (for water) must be adjusted or corrected for the liquid viscosity. This is done using the Hydraulic Institute charts. A typical viscosity corrected performance curve chart is shown in Figure 5.2.


Figure 5.2 Viscosity corrected pump performance

It can be seen that the effect of viscosity is to reduce the head and efficiency at any flow rate compared to that for water. On the other hand the BHP required increases with the viscosity. When selecting a centrifugal pump for a particular application the objective would be to pick a pump that provides the highest efficiency at the desired flow rate. Hence, the pump curve would be selected such as the head and flow requirements are satisfied close to and to the left of the BEP on the pump curve.

Typically, in a centrifugal pump, the pump impeller has a certain diameter but for the same pump case, the impeller may be replaced with a smaller or larger impeller within certain limits. For example, a centrifugal pump may have a rated impeller diameter of 10 inches and produce a certain head versus flow characteristics similar to that depicted in Figure 5.3.


Figure 5.3 Pump performance at different impeller diameters

However, the same pump may be outfitted with an impeller as small as 8 inch diameter or as large as 12 inch diameter Each of these impellers will proportionately reduce or increase the performance characteristics. The performance of a centrifugal pump at different impeller diameters generally follows the Affinity laws. This means that when going from a 10 inch impeller diameter to a 12 inch diameter, the flow and head follow the Affinity Laws as described below.

For an impeller size change from $D_{1}$ to $D_{2}$ :

$$
\begin{array}{ll}
Q_{2} / Q_{1}=D_{2} / D_{1} & \text { for flow rates } \\
H_{2} / H_{1}=\left(D_{2} / D_{1}\right)^{2} & \text { for heads } \tag{5.2}
\end{array}
$$

and

Thus, the flow rate is directly proportional to the impeller diameter, and the head varies directly as the square of the impeller diameter. The efficiency curves at the two different impeller sizes will remain practically the same. Similar to impeller diameter variation, pump speed changes also follow the Affinity laws as follows:

For an impeller speed change from $\mathrm{N}_{1}$ to $\mathrm{N}_{2}$ :

$$
\begin{array}{rll}
Q_{2} / Q_{1}=N_{2} / N_{1} & \text { for flow rates } \\
\text { and } & H_{2} / H_{1}=\left(N_{2} / N_{1}\right)^{2} & \text { for heads } \tag{5.4}
\end{array}
$$

Thus the flow rate is directly proportional to the pump speed while the head is proportional to the square of the pump speed.

The Affinity laws for impeller diameter variation are applicable for small changes in diameter only. However, the Affinity laws can be applied for a wide variation in pump speeds. For example, we may apply the Affinity laws for an impeller size variation from 8 inch to 12 inch with a rated impeller of 10 inch. Extrapolation to 6 inch and 16 inch will not be accurate. On the other hand if the pump is rated at a speed of 3500 rpm , we can apply the Affinity laws for speed variations from 800 rpm to $6,000 \mathrm{rpm}$. However, the higher speeds may not be possible from a design stand point due to high centrifugal forces that are developed at the high speeds.

## Example 11

Using Affinity laws determine the performance of a pump at an impeller size of 11 inch given the following performance at 10 inch impeller diameter:

| Q | 500 | 1000 | 1500 | 2000 |
| :--- | :--- | :--- | :--- | :--- |
| $H$ | 2000 | 1800 | 1400 | 900 |

## Solution

Impeller diameter ratio $=11 / 10=1.10$

The flow rate multiplier is 1.10 and the head multiplier is $1.10 \times 1.10=1.21$
Applying Affinity laws we get the following performance for 11 inch impeller:

| Q | 550 | 1100 | 1650 | 2200 |
| :--- | :--- | :--- | :--- | :--- |
| $H$ | 2420 | 2178 | 1694 | 1089 |

Earlier we briefly discussed the NPSH required for a centrifugal pump. As the flow rate through the pump increases, the NPSH requirement also increases. It is therefore important to calculate the available NPSH based upon the actual piping configuration for a particular pump installation. The calculated NPSH represents the available NPSH and hence must be greater than or equal to the minimum NPSH required for the pump at a particular flow rate. We will illustrate this using the example below.

## Example 12

The bottom of a storage tank containing a liquid ( $\mathrm{Sg}=0.85$, visc $=2.5 \mathrm{cSt}$ ) is located at an elevation of 20 ft above the centerline of the pump as shown in Figure 5.4.


Figure 5.4 NPSH Available

The total length of a 16 inch pipe between the tank and the pump is 180 ft . The liquid level in the tank is 30 ft and the vapor pressure of the liquid at a pumping temperature of $70^{\circ} \mathrm{F}$ is 5 psia. Considering an atmospheric pressure of 14.7 psia, calculate the NPSH available for the pump at a flow rate of $4,000 \mathrm{gal} / \mathrm{min}$.

## Solution

## Pump curve and system curve

Consider a pipeline with a system head curve $A B$ as shown in Figure 5.5


Flow Rate - gal/min

Figure 5.5 Pump curve and system curve - operating point

The pressure required at each flow rate is shown in ft of liquid head. At $2,000 \mathrm{gal} / \mathrm{min}$ the head required is $3,000 \mathrm{ft}$ whereas $4,000 \mathrm{gal} / \mathrm{min}$ requires $4,500 \mathrm{ft}$. A centrifugal pump head curve $C D$ shown superimposed on the system curve intersects the system curve at point $E$ as shown. At E , both the pump head and system head match and are equal to 3,200 ft. If the corresponding flow rate is $2,500 \mathrm{gal} / \mathrm{min}$ we can say that the system requirements and
the pump capability at $2,500 \mathrm{gal} / \mathrm{min}$ are exactly equal. Hence point E represents the operating point when this particular pump is used to pump the liquid through this pipeline. If a lighter product is pumped instead, its system head curve will be lower than the first liquid and hence will intersect the pump head curve at point F which represents a higher flow rate and lower head. Plotting the pump efficiency curve along with the head curve we can also determine the pump efficiency at the operating point and hence the HP required to pump the product.

Suppose the operating point E represents a pressure that exceeds the allowable pipeline pressure. In order to reduce the pressure to that represented by point $G$ we have to reduce the flow rate through the pump to some value $\mathrm{Q}_{1}$. This is accomplished by using a control valve on the discharge of the pump. The reduced flow rate causes a difference in pump head curve and system head curve equal to the length GK. This head represents the throttle pressure at the flow rate $\mathrm{Q}_{1}$. The pump case pressure is the pressure developed by the pump at flow rate $\mathrm{Q}_{1}$ and represented by point K on the pump head curve. Since the throttle pressure is the amount of pump head wasted (GK), a certain amount of pump HP is wasted as well. This HP lost due to throttling can be calculated easily knowing the flow rate $\mathrm{Q}_{1}$, the throttle pressure and the pump efficiency at $\mathrm{Q}_{1}$. HP lost due to throttling at flow rate $Q_{1}$ is as follows:

$$
\begin{equation*}
\mathrm{HP}=\mathrm{Q} 1 \times \mathrm{H}_{\mathrm{GK}} \times \mathrm{Sg} /(3960 \times \text { Effy }) \tag{5.5}
\end{equation*}
$$

If the pump was driven by a variable speed motor we could lower the pump speed and match the system head required at flow rate $\mathrm{Q}_{1}$. This is shown as a dotted head curve in Figure 5.5. Obviously, with a variable speed pump, throttling is eliminated and therefore no HP is lost. The system head curve requirement at point $G$ at a flow rate of $Q_{1}$ is exactly matched by the dotted pump head curve at the reduced speed.

## Example 13

Calculate the HP lost in throttling when the operating point is moved from point E to point K in Figure 5.5 If the original pump speed were $3,500 \mathrm{rpm}$, at what reduced speed using a variable speed drive must the pump be run to minimize throttle pressure.

## Solution

Suppose the flow rate corresponding to the point $K$ on the pump head curve is 2,300 $\mathrm{gal} / \mathrm{min}$, head is $3,500 \mathrm{ft}$ and efficiency is $78 \%$. The system head at G is $3,100 \mathrm{ft}$. Therefore, the throttled head $=3500-3100=400 \mathrm{ft}$

The HP lost in throttling is:
HP lost $=2300 \times 400 \times 1.0 /(3960 \times 0.78)=298$, considering water.

If the motor efficiency is $95 \%$ and the electrical energy cost is 10 cents/KWH, assuming 24 hour a day operation, 350 days per year, the energy cost attributed to the throttling is:
$298 \times 0.746 \times 24 \times 350 \times 0.10 / 0.95=\$ 204,991$ per year

In order to determine the reduced pump speed at which the system head requirement ( $3,100 \mathrm{ft}$ at $2300 \mathrm{gal} / \mathrm{min}$ ) matches the pump curve, we can create pump head curves at various speeds below 3,500 rpm using Affinity laws. Thus the reduced speed head curve that matches the system head requirement can be found.

An approximate reduced speed can be calculated from:

$$
(N 2 / 3500)^{2}=(3100 / 3200)
$$

Solving for N2, we get:

$$
\mathrm{N} 2=3,445 \mathrm{rpm}
$$

This is only an approximate value. We must plot the pump head curves at various speeds in the vicinity of $3,445 \mathrm{rpm}$ (say $3,400-3,500 \mathrm{rpm}$ ) to find the correct speed.

## 6. Drag Reduction Effect

We have seen that the pressure drop due to friction at a certain flow rate depends upon the pipe diameter, pipe roughness and liquid properties. It has been found that continuously injecting small amounts (in parts per million or ppm) of certain hydrocarbons in the liquid being pumped results in lower pressure drops as long as the flow is turbulent. This is referred to as drag reduction and the product injected is called the drag reduction agent (DRA). The effectiveness of DRA decreases as the treated liquid flows through pipeline restrictions such as valves, meters and pump stations. The mechanism of drag reduction is very complex and is not fully understood. Suffice it to say that under certain conditions as long as flow is turbulent and the viscosity of the pumped liquid is not too high, lower friction drop ( $\mathrm{psi} / \mathrm{mi}$ ) is experienced at the same flow rate when DRA is injected into the pipeline.

As an illustration, in a previous example we found in a NPS 20 pipeline transporting diesel at $8,000 \mathrm{bbl} / \mathrm{h}$ the pressure drop due to friction was $12.72 \mathrm{psi} / \mathrm{mi}$. By injecting 10 ppm of a certain DRA we may be able to obtain $20 \%$ drag reduction. This means that the pressure drop will reduce from $12.72 \mathrm{psi} / \mathrm{mi}$ to $10.18 \mathrm{psi} / \mathrm{mi}$ (a $20 \%$ drop). This reduction will cause a reduction in the pressure required to pump diesel at the same flow rate. This in turn will also reduce the actual HP required to pump the liquid. However, the application of DRA is to increase pipeline flow rate by a certain amount by increasing the pressure drop back to the original value of $12.72 \mathrm{psi} / \mathrm{mi}$ with DRA. This is illustrated in Figure 6.1.


Flow Rate

Figure 6.1 Drag Reduction effect on system head curve

Without DRA, the operating point for the pipeline system curve and the pump curve is shown as the point E . With DRA injection, the system curve slides to the right and below as shown in the dashed system curve. The operating point thus shifts to the point G . At G , the flow rate is higher than at E. However, at the increased flow, the pump HP required will also increase. The drive HP must also be adequate to operate at this increased flow rate.

The quantity of DRA injected is in ppm. For example, if the pipeline flow rate is $8,000 \mathrm{bbl} / \mathrm{h}$ and we are using 10 ppm of DRA, the actual DRA injection rate can be calculated as follows:

$$
\text { DRA ppm }=10=\left(10 / 10^{6}\right) \times 8000=0.08 \mathrm{bbl} / \mathrm{h}=0.08 \times 42 \times 24 \mathrm{gal} / \text { day }
$$

or $\operatorname{DRA}$ injection rate $=80.64 \mathrm{gal} /$ day

Therefore, for 10 ppm , we need to continuously inject DRA at the rate of $80.64 \mathrm{gal} /$ day to achieve the desired drag reduction.

One application of drag reduction is in a pipeline with multiple pump stations, which has a bottleneck segment. Suppose there are three pump stations with two stations discharging below MAOP while the third pump station discharges at MAOP as shown in Figure 6.2.


Figure 6.2 Pipeline bottleneck and DRA application

Due to this MAOP limit in the segment $A B$, the pipeline throughput is limited to some value Q. If we increase the flow rate above Q , the discharge pressure at the critical pump station (A) will exceed MAOP. Since this is not allowable, DRA can be injected into the bottleneck pump station segment immediately downstream of $A$ to reduce friction and hence the discharge pressure as the flow rate is increased. Since the effectiveness of the DRA diminishes from pump station to pump station, the other two pump stations ( $B$ and $C$ ) will operate at higher discharge pressures than before at the increased flow rate. This may be permissible to some extent since originally, these two pump stations were operating below MAOP and there is still room left for the discharge pressure to reach the MAOP limit as indicated by the dashed lines in the hydraulic gradient in Figure 6.2.

Once these two pump stations reach the MAOP limit, at some flow rate higher than Q , these two segments $B C$ and $C D$ become the bottleneck segments. Any further increase in pipeline throughput is possible only by using DRA in these two segments as well. Note that with DRA injection in segment $A B$ at the higher flow rate, the hydraulic gradient for $A B$ will be different from that of segment $B C$ and $C D$, since the DRA effectiveness is only in segment $A B$. As the DRA treated liquid flows through pump stations $B$ and $C$, the DRA effectiveness is lost due to the degradation of DRA.

The amount of DRA required in ppm to achieve a certain flow increase depends upon many factors. For a particular pipeline, depending upon the liquid viscosity, gravity and the Reynolds number, DRA effectiveness varies with flow rate. DRA vendors such as Baker Petrolite and Conoco-Phillips have their own algorithms to calculate the percentage drag reduction possible for a given DRA injection rate. In general, the percentage drag reduction will increase with ppm of DRA up to some point known as maximum drag reduction (MDR). For further details on this matter, contact one of the DRA vendors.

## 7. Batching different products

So far in our hydraulic analysis we considered a single product flowing through a pipeline at any time. In this section under flow injection and deliveries we also looked at two streams of liquid commingled and transported through the pipeline as a single product. When commingling of products is not acceptable, but multiple products have to be transported simultaneously in a pipeline, we have what is known as batching. Consider a pipeline 100 mi long, that has a line fill (total volume) capacity of 150,000 bbl. In a typical batching situation, 80,000 bbl batch of gasoline will be followed by a 30,000 bbl batch of diesel and 40,000 bbl batch of jet fuel as shown in Figure 7.1


Figure 7.1 Batched pipeline

At the boundary of any two products there will be some commingling which will have to be separated as the products reach the pipeline terminus. Generally, the commingled liquid is switched to a slop tank while the pure products of gasoline, diesel and jet fuel will be diverted to their respective tanks at the pipeline destination. It can be seen from Figure 7.1 that at any instant in the batched movement of products, the flow rate versus pressure drop will depend upon the frictional pressure drops for each liquid that occupies a certain section of the pipeline. If gasoline occupied the entire pipeline, the pressure drop in the 100 mi pipeline segment will be calculated considering the specific gravity and viscosity of gasoline. Being the lightest product of the three, the pressure drop at any flow rate will be the lowest for gasoline. In comparison, if diesel filled the pipeline completely, the pressure drop will be the highest at the same flow rate. Finally, with jet fuel filling the entire pipeline, the
pressure drop at the same flow rate will be a value between those for gasoline and diesel. If we were to calculate separately the maximum flow rate possible for each product within the MAOP limit of the pipeline, we may come up with the following flow rates:

$$
\begin{array}{ll}
\mathrm{Q}_{\max }=8,700 \mathrm{bbl} / \mathrm{h} & \text { for gasoline } \\
\mathrm{Q}_{\max }=6,500 \mathrm{bbl} / \mathrm{h} & \text { for jet fuel } \\
\mathrm{Q}_{\max }=5,200 \mathrm{bbl} / \mathrm{h} & \text { for diesel }
\end{array}
$$

In the batched configuration the maximum flow rate possible will depend upon the amounts of each products filling the pipeline. As a rough approximation we could use a weighted average depending upon the batch volumes. If each product occupies a third of the pipeline at any instant, the maximum flow rate in the batched configuration is approximately:

$$
\mathrm{Q}_{\max }=(8700+6500+5200) / 3=6,800 \mathrm{bbl} / \mathrm{h}
$$

Of course, this is a simplistic approach to determining batched hydraulics of pipelines, and a more accurate method is required to determine the flow rate possible in a batched pipeline. Before we pursue this more accurate approach it will be instructive to determine the approximate flow rate with the batched volumes defined earlier, shown in Figure 7.1.

The weighted average flow rate in this case can be calculated as follows:
Fraction of line fill for gasoline $=80000 / 150000=0.5333$
Fraction of line fill for jet fuel $=40000 / 150000=0.2667$
Fraction of line fill for diesel $=1-(0.5333+0.2667)=0.20$
Therefore:

$$
\mathrm{Q}_{\max }=(8700 \times 0.5333)+(6500 \times 0.2667)+(5200 \times 0.20)=7,413 \mathrm{bbl} / \mathrm{h}
$$

## 8. Pipe Analysis

In the previous sections, we calculated the pressure needed to pump a certain flow rate of liquid through a pipeline. We also briefly discussed that if the pressure required exceeds the maximum allowable operating pressure (MAOP) of the pipeline, we will need to install additional booster pump stations to provide the total pressure to pump the liquid, without exceeding MAOP. The MAOP of a pipeline is the maximum safe internal pressure that the pipeline can withstand without failure. This will depend upon the pipe material, diameter and wall thickness. As the internal pressure is increased, the pipe material gets stressed more and more and, ultimately, will undergo permanent deformation and will rupture. Therefore it is very important to determine the safe internal pressure that the pipe can withstand, without rupture, so that we may operate the pipeline safely.

The Barlow's equation in a modified form is used to calculate this allowable internal pressure as follows:

In USCS units, the internal design pressure in a pipe is calculated as follows:

$$
\begin{equation*}
P=(2 T \times S \times E \times F) / D \tag{8.1}
\end{equation*}
$$

where $P$ is the internal design pressure in psig. The pipe outside diameter $D$ and wall thickness $T$ are in inches. $S$ is the Specified Minimum Yield Strength (SMYS) of pipe material in psig and E is the Seam Joint Factor, 1.0, for seamless and Submerged Arc Welded (SAW) pipes. $F$ is the Design Factor, usually 0.72 , for liquid pipelines except that a design factor of 0.60 is used for pipes, including risers, on a platform located off shore or on a platform in inland navigable waters. Similarly, a design factor of 0.54 is used for pipes that have been subjected to cold expansion to meet the SMYS and subsequently heated, other than by welding or stress relieving as a part of the welding, to a temperature higher than $900^{\circ} \mathrm{F}$ $\left(482^{\circ} \mathrm{C}\right)$ for any period of time or over $600^{\circ} \mathrm{F}\left(316^{\circ} \mathrm{C}\right)$ for more than one hour.

The above version of Barlow's equation is described in Part 195 of US DOT code of Federal Regulations, Title 49 and ASME/ANSI standard B31.4 for liquid pipelines.

In SI units, the internal design pressure equation can be written as:

$$
\begin{equation*}
P=(2 T \times S \times E \times F) / D \tag{8.2}
\end{equation*}
$$

where $P$ is the pipe internal design pressure in $k P a$. The pipe outside diameter $D$ and wall thickness T are in mm. S is the Specified Minimum Yield Strength (SMYS) of the pipe material in kPa , and E and F are the same as before.

Once we know the safe internal working pressure for the pipe, a hydrostatic test pressure is established. Generally, for liquid pipelines this test pressure is $125 \%$ of the allowable working pressure. The hydrostatic test pressure is the pressure at which the pipeline is tested for a specified period of time, usually 4 to 8 hours, depending upon the design code. For above ground piping 4 hours is used, while buried pipelines are tested for 8 hours. In the USA, the Department of Transportation (DOT) Code Part 195 applies for liquid pipelines. Therefore, if the MAOP is 1,000 psig, the hydrotest pressure will be $1.25 \times 1000=1250 \mathrm{psig}$.

The SMYS of the pipe material depends on the type of steel used in constructing the pipe. The API 5L standard for pipe covers many grades: $\mathrm{X}-42, \mathrm{X}-52, \mathrm{X}-60, \mathrm{X}-65, \mathrm{X}-70$ and $\mathrm{X}-80$. The number after the X represents the SMYS in thousands of psi. Thus $\mathrm{X}-60$ pipe has an SMYS of 60,000 psi.

For example, the MAOP for a 16 inch pipeline, 0.250 inch wall, constructed of $X-52$ pipe material is:
$P=(2 \times 0.250 \times 52000 \times 1.0 \times 0.72) / 16=1,170 \mathrm{psig}$
The hydrotest pressure is therefore:
$1.25 \times 1170=1,463 \mathrm{psig}$

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